

宿題 ( 1 ) ( 提出日: 1 1 月 2 2 日 )

[1]

$$\begin{aligned}
 v &= \frac{Q}{\pi R^2} = \frac{1}{\pi R^2} \int_0^R 2\pi(R-y)u dr \\
 &= \frac{1}{\pi R^2} \int_R^0 2\pi(R-y)u dy; \quad u = U + \frac{u^*}{\kappa} \ln \frac{y}{R} \\
 &= \frac{2\pi}{\pi R^2} \int_R^0 (U + \frac{u^*}{\kappa} \ln \frac{y}{R})(R-y) dy \\
 &= 2 \int_1^0 (U + \frac{u^*}{\kappa} \ln \frac{y}{R})(1 - \frac{y}{R}) d(\frac{y}{R}); \quad \frac{y}{R} = \eta \\
 &= 2 \int_1^0 (U + \frac{u^*}{\kappa} \ln \eta)(1 - \eta) d\eta \\
 &= 2 \int_1^0 (U + \frac{u^*}{\kappa} \ln \eta - U\eta - \frac{u^*}{\kappa} \eta \ln \eta) d\eta \\
 &= 2[U\eta + \frac{u^*}{\kappa}(\eta \ln \eta - \eta) - \frac{U}{2}\eta^2 - \frac{u^*}{\kappa}(\ln \eta \frac{\eta^2}{2} - \frac{\eta^2}{4})]_1^0 \\
 &= 2[U - \frac{U}{2} - \frac{u^*}{\kappa} + \frac{u^*}{\kappa} \frac{1}{4}] \\
 &= U - \frac{3}{2} \frac{u^*}{\kappa} = U - 3.75u^*(\kappa = 0.4) \\
 (\int \ln x dx &= x \ln x - x; \quad \int x \ln x dx = \int \ln x d(\frac{x^2}{2}) dx = \ln x \frac{x^2}{2} - \int \frac{1}{x} \frac{x^2}{2} dx = \ln x \frac{x^2}{2} - \frac{x^2}{4}
 \end{aligned}$$

[2]

$$\begin{aligned}
 \frac{\tau}{\tau_w} &= \frac{r}{R} = (1 - \frac{y}{R}) \\
 \tau &= \rho l^2 (\frac{du}{dy})^2 = \tau_w (1 - \frac{y}{R}) \\
 l &= \frac{u^* \sqrt{1 - y/R}}{du/dy} \\
 \frac{u}{U} &= (\frac{y}{R})^{1/7}; \quad \frac{du}{dy} = \frac{U}{R} \frac{1}{7} (\frac{y}{R})^{-6/7} \\
 \frac{l}{R} &= \frac{u^*}{U} 7 (\frac{y}{R})^{6/7} \sqrt{1 - y/R}
 \end{aligned}$$