理想流体力学演習問題1-1

Example 4-1(White, p194)
Given the eulerian velocity-vector field

$$V = 3ti + xzj + ty^2k$$

find the acceleration of a particle.
(Solution)

$$u = 3t, \quad v = xz, \quad w = ty^2$$
$$\frac{\partial V}{\partial t} = i\frac{\partial u}{\partial t} + j\frac{\partial v}{\partial t} + k\frac{\partial w}{\partial t} = 3i + y^2k$$
$$\frac{\partial V}{\partial x} = zj, \quad \frac{\partial V}{\partial y} = 2tyk, \quad \frac{\partial V}{\partial z} = xj$$
$$\frac{dV}{dt} = (3i + y^2) + 3tzj + 2txyzk + txy^2j$$
$$\therefore \quad \frac{dV}{dt} = 3i + (3tz + txy^2)j + (2txyzy^2)k$$

Example 4.3
An incompressible velocity field is given by

$$u = a(x^2 - y^2) \quad v \quad \text{unknown} \quad w = b$$

where a and b are constant. What must the form of the velocity component v be?
(Solution)

Again Eq. (4.2a) applies

$$\frac{\partial}{\partial x}(ax^2 - ay^2) + \frac{\partial v}{\partial y} + \frac{\partial b}{\partial z} = 0$$
$$\frac{\partial v}{\partial y} = -2ax$$

This is easily integrated partially with respect to y

$$v(x, y, z, t) = -2axy + f(x, z, t)$$

This is the only possible form for v which satisfies the incompressible continuity equation. The function of integration f is entirely arbitrary since it vanishes when v is differentiated with respect to y.

Example 4.7
If a stream function exists for the velocity field of Example 4.5

$$u = a(x^2 - y^2) \quad v = -2axy \quad w = 0$$

find it, plot it, and interpret it.
Solution:

Since this flow field was shown expressly in Example 4.3 to satisfy the equation of continuity, we are pretty sure that a stream function does exist. We can check again to see if

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Substitute:

$$2ax + (-2ax) = 0$$

checks

Therefore we are certain that a stream function exists. To find ψ, we simply set

$$u = \frac{\partial \psi}{\partial y} = ax^2 - ay^2$$

$$v = -\frac{\partial \psi}{\partial x} = -2axy$$

and work from either one toward the other. Integrate (1) partially

$$\psi = ax^2y - \frac{ay^2}{3} + f(x)$$

Differentiate (3) with respect to x and compare with (2)

$$\frac{\partial \psi}{\partial x} = 2axy + f'(x) = 2axy$$

Therefore $f'(x) = 0$, or $f = constant$. The complete stream function is fhus found

$$\psi = a(x^2y - \frac{y^3}{3} + C)$$

To plot this, set C=0 for convenience and plot the function

$$3x^2y - y^2 = \frac{3\psi}{a}$$

for constant values of ψ. The result in shown in Fig. E4.7a to be six 60^o wedges of circulating motion, each with identical flow patterns except for the arrows. Once the streamlines are labeled, the flow directions follow from the sign convention of Fig. 4.9.How can the flow be interpreted? Since there is slip along all streamlines, no streamline can truly represent a solid surface in a viscous flow. However, the flow could represent the impingement of three incoming streams at 60, 180, and 300^o. This would be a rather unrealistic yet exact solution to the Navier-Stokes equation, as we showed in Example 4.5.

By allowing the flow to slip as a frictionless approximation, we could let any given streamline be a body shape. Some examples are shown in Fig. E4.7b.