理想流体力学試験問題

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1. (35) 複素ポテンシャルが次式で表される流れの型を説明し、かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ.

$$(1)\ w = aze^{i\alpha}\ (\alpha > 0),\ (2)\ w = z^n\ (n = \frac{2}{3})$$

$$(3)\ w = -5i\ln z + 3z,\ (4)\ w = 3z + 2\ln z$$

2. (25) (1) 二次元の渦流れにおいて，速度成分が $u = 4y,\ v = 2x$ なる流れは理論上存在しうるか．(2) その流れの流線を求めよ．(3) 直線 $y = 1,\ y = 3,\ x = 2,\ x = 5$ で区切られた長方形のまわりの循環値を求めよ．3. (25) 速度 U の一様流れ中に強さ Q の吹き出しが原点にある場合、この流れ場に作用する力を求めよ．4. (25) $4a$ の長さの平板に α なる傾きをもち，かつ循環をもつ流れがある．(1) 流れの複素ポテンシャルを求めよ．(2) 平行流れ (w-平面) から平板に至る写像関係を示し，かつ流れをスケッチせよ．(3) 平板の後端に岐点がくるようにしたときの循環値をを求めよ．
5. (25) 二次元の渦流れで，その速度成分が $v_r = 0,\ v_\theta = \omega r$ なるときの渦度を求めよ．6. (25) 速度成分が $u = ax + by,\ v = cx + dy$ で示される流れが非圧縮性流体となるための条件を示せ．また，流れが渦なし流れとした場合の流れ関数を求めよ．

（解）

1.

(1) Parallel flow with $\theta = \alpha$

$w = ar\{(\cos(\theta + \alpha) + i\sin(\theta + \alpha)\}$

$\varphi = ar\cos(\theta + \alpha), \quad \psi = ar\sin(\theta + \alpha)$

$\frac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i\sin\alpha) = u - iv$

$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$

(2) Corner flow with $\theta = \frac{3}{2}\pi$

$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$

$\varphi = r^n\cos n\theta, \quad \psi = r^n\sin n\theta$

$For\ n = \frac{2}{3}, \quad \varphi = r^{2/3}\cos\frac{2\theta}{3}, \quad \psi = r^{2/3}\sin\frac{2\theta}{3}$

(3) Parallel flow(U=3)+Circulation flow($\Gamma = 10\pi$)

$w = -5i\ln(re^{i\theta}) + 5re^{i\theta} = -5i\ln r + 5\theta + 3r(\cos\theta + i\sin\theta)$

$\varphi = 5\theta + 3r\cos\theta, \quad \psi = 3r\sin\theta - 5\ln r$

(4) Parallel flow(U=3)+source flow($Q = 4\pi$)

$w = 3re^{i\theta} + 2\ln(re^{i\theta})$

$\varphi = 3r\cos\theta + 2\ln r, \quad \psi = 3r\sin\theta + 2\theta$

2.

$$
\begin{aligned}
&(1)\ divV = 0\\
&(2)\ \frac{dx}{4y} = \frac{dy}{2x}, \quad 2xdx - 4ydy = 0, \quad x^2 - 2y^2 = c\\
&(3)\ 4(2-5) + 10(1-3) - 12(1-5) - 4(3-1) = -12m^2/s\\
&\Gamma = \int_2^5 \int_1^3 (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})dxdy\\
&= -\int_1^3 6dy = -(18-6) = -12m^2/s
\end{aligned}
$$

3.

$$
\begin{aligned}
&w = Uz + m\ln z, \quad m = \frac{Q}{2\pi}\\
&\frac{dw}{dz} = U + \frac{m}{z}\\
&(\frac{dw}{dz})^2 = U^2 + \frac{m^2}{z^2} + \frac{2Um}{z}\\
&F_x - iF_y = \frac{i\rho}{2}\oint(\frac{dw}{dz})^2 dz = \frac{i\rho}{2}2Um(2\pi i) = -2\pi\rho Um = -\rho UQ\\
&F_x = -\rho UQ, \quad F_y = 0
\end{aligned}
$$

4.

$$
\begin{aligned}
&w = U(z_1 + \frac{a^2}{z_1}) - \frac{i\Gamma}{2\pi}\ln z_1, \quad z_2 = z_1 e^{i\alpha}, \quad z = z_2 + \frac{a^2}{z_2}\\
&\frac{dw}{dz_1}\frac{dz_1}{dz_2}\frac{dz_2}{dz} = 0\\
&\frac{dw}{dz_1})_A = U(1 - \frac{a^2}{z_1^2}) - \frac{i\Gamma}{2\pi z_1} = 0\\
&At\ point\ A,\ z = 2a,\ z_2 = a, \quad z_1 = z_2 e^{-i\alpha} = ae^{-i\alpha}\\
&\frac{dw}{dz_1})_A = U(1 - \frac{a^2}{a^2 e^{-2i\alpha}}) - \frac{i\Gamma}{2\pi a e^{-i\alpha}} = 0\\
&U(1 - e^{2i\alpha}) - \frac{i\Gamma}{2\pi a}e^{i\alpha} = 0\\
&U(e^{-i\alpha} - e^{i\alpha}) - \frac{i\Gamma}{2\pi a} = 0\\
&U(\cos\alpha - i\sin\alpha - \cos\alpha - i\sin\alpha) - \frac{i\Gamma}{2\pi a} = 0\\
&\Gamma = -4\pi a U\sin\alpha\ (\Gamma:\ negative)
\end{aligned}
$$

5.

$$
\begin{aligned}
&v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 0, \quad \psi = f(r)\\
&v_\theta = -\frac{\partial\psi}{\partial r} = \omega r, \quad \psi = -\frac{1}{2}\omega r^2 + f(\theta)\\
&\psi = -\frac{1}{2}\omega r^2 = -\frac{1}{2}\omega(x^2 + y^2)\\
&\zeta = -\nabla^2\psi = -(\omega - \omega) = 2\omega
\end{aligned}
$$

6.

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0
$$

$$u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy$$

$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$

$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$

For irrotational flow, $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$, $b = c$, $\psi = axy + \dfrac{b}{2}(y^2 - x^2) + const.$