理想流体力学 試験問題

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1. (20) 速度成分が $u = ax + by,\ v = cx + dy$ で示される流れが非圧縮性流体となるための条件を示せ．また，流れが渦なし流れとした場合の流れ関数を求めよ．2. (30) 複素ポテンシャルが次式で表される流れの型を説明し、かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ．

(1) $w = aze^{i\alpha}$ $(\alpha > 0)$, (2) $w = z^n$ $(n = \dfrac{1}{2})$, (3) $w = -5i \ln z + 3z$, (4) $w = 2z + 3 \ln z$

3. (20) 速度 U の一様流れ中に強さ Q の吹き出しが原点にある場合、この流れ場に作用する力を求めよ．

4. (20) (1) 二次元の渦流れにおいて，速度成分が $u = 4y,\ v = 2x$ なる流れは理論上存在しうるか．(2) その流れの流線を求めよ．(3) 直線 $y = 1,\ y = 3,\ x = 2,\ x = 5$ で区切られた長方形のまわりの循環値を求めよ．

5. (20) 図に示すような流線図より, この流れはどういう型の流れを組み合わせたものかを説明せよ. また数値も含めた複素ポテンシャルを求めよ.

（解）

1.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0$$

$$u = \frac{\partial \psi}{\partial y} = ax + by, \quad v = -\frac{\partial \psi}{\partial x} = cx + dy$$

$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$

$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$

For irrotational flow, $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x},\ b = c,\ \psi = axy + \dfrac{b}{2}(y^2 - x^2) + const.$

2.

(1) Parallel flow with $\theta = \alpha$

$w = ar\{(\cos(\theta + \alpha) + i \sin(\theta + \alpha)\}$

$\varphi = ar \cos(\theta + \alpha), \quad \psi = ar \sin(\theta + \alpha)$

$\dfrac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i \sin\alpha) = u - iv$

$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$

(2) Corner flow with $\theta = 2\pi$

$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i \sin n\theta)$

$\varphi = r^n \cos n\theta, \quad \psi = r^n \sin n\theta$

$For\ n = \dfrac{1}{2}, \quad \varphi = r^{1/2} \cos\dfrac{\theta}{2}, \quad \psi = r^{1/2} \sin\dfrac{\theta}{2}$

(3) Parallel (U=3)+circulation($\Gamma = 10\pi$) flow

$w = -5i\ln(re^{i\theta}) + 3re^{i\theta} = -5\ln r + 5\theta + 3r(\cos\theta + i\sin\theta)$

$\varphi = 5\theta + 3r\cos\theta, \quad \psi = 3r\sin\theta - 5\ln r$

(4) Parallel flow(U=2)+source flow($Q = 6\pi$)

$w = 2re^{i\theta} + 3\ln(re^{i\theta})$

$\varphi = 2r\cos\theta + 3\ln r, \quad \psi = 2r\sin\theta + 3\theta$

3.

$w = Uz + m\ln z, \quad m = \dfrac{Q}{2\pi}$

$\dfrac{dw}{dz} = U + \dfrac{m}{z}$

$(\dfrac{dw}{dz})^2 = U^2 + \dfrac{m^2}{z^2} + \dfrac{2Um}{z}$

$F_x - iF_y = \dfrac{i\rho}{2}\oint(\dfrac{dw}{dz})^2 dz = \dfrac{i\rho}{2}2Um(2\pi i)$

$F_x = -\rho UQ, \quad F_y = 0$

4.

(1) $divV = 0$

(2) $\dfrac{dx}{4y} = \dfrac{dy}{2x}, \quad 2xdx - 4ydy = 0, \quad x^2 - 2y^2 = c$

(3) $4(5-2) + 10(3-1) - 12(5-1) - 4(1-3) = 12m^2/s$

$\Gamma = \displaystyle\int_2^5\int_1^3 (\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y})dxdy$

$= -\displaystyle\int_1^3 6dy = -(18-6) = -12m^2/s$

5.

$w = iUz + m\ln\dfrac{z - z_2}{z - z_1},\ z_1 = 0,\ z_2 = 3 + 4i$

$U = 4m/s,\ m = \dfrac{Q}{2\pi} = \dfrac{27\times 1\times 4}{2\pi} = \dfrac{54}{\pi}$

$w = i4z + \dfrac{54}{\pi}\ln[1 - \dfrac{3+4i}{z}]$