流体力学III試験問題

1987-9-22

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1. 複素ポテンシャルが $w = -ilnz + 2z$ で与えられる流れについて：
(1) これはどういう型の流れを組み合わせたものか
(2)Potential function, Stream function を求めよ
(3)Stagnation point(or points) を求めよ
(4)$r = 1,\ \ \theta = \frac{3}{2}\pi$ にこける速度を求めよ。
2. 速度成分が $u = ax + by$, $v = cx + dy$ で示される流れが非圧縮性流体となるための条件を示せ．また，流れが渦なし流れとした場合の流れ関数を求めよ．
3. 複素ポテンシャルが次式で表される流れの型を説明し、かつそれらの流れの速度ポテンシャルおよび流れの関数を求めよ．また、(1) の速度成分、u,v および合速度 V を求めよ。

$$(1)\ w = aze^{i\alpha}\ (\alpha > 0),\ (2)\ w = z^n\ (n = \frac{2}{3})$$

4. 二次元非圧縮性流体の連続の式を極座標で表すと次のようになる。いま、特別な流れとして $v_r = -\mu\cos\theta/r^2$ で示される流れの v_θ および合速度を求めよ。

$$\frac{\partial(v_r r)}{\partial r} + \frac{\partial v_\theta}{\partial\theta} = 0$$

（解）

1.

(1) $\quad Circulation + parallel\ flow$

(2) $\quad w = -i\ln(re^{i\theta}) + 2re^{i\theta} = -i\ln r + \theta + 2r(\cos\theta + i\sin\theta)$

$$= (\theta + 2r\cos\theta) + i(2r\sin\theta - \ln r)$$

$$\varphi = \theta + 2r\cos\theta, \quad \psi = 2r\sin\theta - \ln r$$

(3) $\quad \dfrac{dw}{dz} = -\dfrac{i}{z} + 2 = 2 - i\dfrac{1}{r}(\cos\theta - i\sin\theta) = 0$

$$z = \frac{i}{2} = x + iy \quad x = 0 \quad y = \frac{1}{2}$$

(4) $\quad$ At $r = 1, \quad \theta = \dfrac{3\pi}{2}; \quad \dfrac{dw}{dz} = 2 - i\{0 - i(-1)\} = 3, \quad V = 3$

2.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad a + d = 0$$

$$u = \frac{\partial\psi}{\partial y} = ax + by, \quad v = -\frac{\partial\psi}{\partial x} = cx + dy$$

$$\psi = axy + \frac{b}{2}y^2 + f(x), \quad \psi = -\frac{c}{2}x^2 - dxy + f(y) = axy - \frac{c}{2}x^2 + f(y)$$

$$\psi = axy + \frac{1}{2}(by^2 - cx^2) + const.$$

For irrotational flow, $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x},\ b = c,\ \psi = axy + \dfrac{b}{2}(y^2 - x^2) + const.$

3. （解）

(1) Parallel flow with $\theta = \alpha$

$$w = ar\{(\cos(\theta+\alpha) + i\sin(\theta+\alpha)\}$$

$$\varphi = ar\cos(\theta+\alpha), \quad \psi = ar\sin(\theta+\alpha)$$

(2) Corner flow with $\theta = \frac{3}{2}\pi$

$$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$

$$\varphi = r^n \cos n\theta, \quad \psi = r^n \sin n\theta$$

$$For\ n = \frac{2}{3}, \quad \varphi = r^{2/3}\cos\frac{2\theta}{3}, \quad \psi = r^{2/3}\sin\frac{2\theta}{3}$$

$$\frac{dw}{dz} = ae^{i\alpha} = a(\cos\alpha + i\sin\alpha) = u - iv$$

$$u = a\cos\alpha, \quad v = -a\sin\alpha, \quad V = a$$

4.

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 0; \psi = f(r)$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = \omega; \psi = -1/2\omega r^2$$

$$\psi = -1/2\omega(x^2 + y^2), r^2 = x^2 + y^2$$

$$\therefore \quad \zeta = -\Lambda^2\psi = -2\omega$$

Another solution

$$u = v_\theta sin\theta = \omega r sin\theta = \omega y; \quad v = -v_\theta cos\theta = \omega x$$

$$\therefore \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\omega - \omega = -2\omega$$