流体力学III試験問題

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1. 複素ポテンシャルが $w = cz^n$(c は常数) で示される流れで、n=1, 2/3, -1 の場合の流れについてそれぞれ説明せよ。

2. 次の流れを説明し、これらはすべて理論上存在しうる流であり、かつ (4) 以外はすべてうずなし流れであることを示せ。

$$(1)\ \psi = 15y, \quad (2)\ \psi = 17.3y - 10x, \quad (3)\ \psi = -20x, \quad (4)\ \psi = -5x^2$$

3. 複素ポテンシャルが $w = -ilnz + 2z$ で与えられる流れについて：

(1) これはどういう型の流れを組み合わせたものか

(2)Potential function, Stream function を求めよ

(3)Stagnation point(or points) を求めよ

(4)$r = 1$, $\theta = \frac{3}{2}\pi$ にこける速度を求めよ。

(解)

1.

(1) $$\frac{dw}{dz} = ae^{-\alpha} = a(\cos\alpha - i\sin\alpha) = u - v$$
$$u = a\cos\alpha, \quad v = a\sin\alpha, \quad V = a$$

(2) $$z = re^{i\theta}, \quad w = \varphi + i\psi = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$
$$\varphi = r^n\cos n\theta, \quad \psi = r^n\sin\theta$$
$$For\ n = \frac{1}{2}, \quad \varphi = r^{1/2}\cos\frac{\theta}{2}, \quad \psi = r^{1/2}\sin\frac{\theta}{2}$$

2.

(1) $$u = \frac{\partial\psi}{\partial y} = 15, \quad v = \frac{\partial\psi}{\partial x} = 0, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

(2) $$u = \frac{\partial\psi}{\partial y} = 17.3, \quad v = \frac{\partial\psi}{\partial x} = 10, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

(3) $$u = \frac{\partial\psi}{\partial y} = 0, \quad v = \frac{\partial\psi}{\partial x} = 20, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

(4) $$u = \frac{\partial\psi}{\partial y} = 0, \quad v = \frac{\partial\psi}{\partial x} = 10x, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 10(\text{rotational})$$

3.

(1) $$Circulation + parallel\ flow$$

(2) $$w = -i\ln(re^{i\theta}) + 2re^{i\theta} = -i\ln r + \theta + 2r(\cos\theta + i\sin\theta)$$
$$= (\theta + 2r\cos\theta) + i(2r\sin\theta - \ln r)$$
$$\varphi = \theta + 2r\cos\theta, \quad \psi = 2r\sin\theta - \ln r$$

(3) $$\frac{dw}{dz} = -\frac{i}{z} + 2 = 2 - i\frac{1}{r}(\cos\theta - i\sin\theta) = 0$$
$$z = \frac{i}{2} = x + iy \quad x = 0 \quad y = \frac{1}{2}$$

(4) $$\text{At } r = 1, \quad \theta = \frac{3\pi}{2}; \quad \frac{dw}{dz} = 2 - i\{0 - i(-1)\} = 3, \quad V = 3$$