

宿題（２）（提出日：1 1月2 9日）

[1]

$$\begin{aligned}\frac{\tau}{\tau_w} &= \frac{r}{R} = \left(1 - \frac{y}{R}\right) \\ \tau &= \rho l^2 \left(\frac{du}{dy}\right)^2 = \tau_w \left(1 - \frac{y}{R}\right) \\ l &= \frac{u^* \sqrt{1 - y/R}}{du/dy} \\ \frac{u}{U} &= \left(\frac{y}{R}\right)^{1/7}; \quad \frac{du}{dy} = \frac{U}{R} \frac{1}{7} \left(\frac{y}{R}\right)^{-6/7} \\ \frac{l}{R} &= \frac{u^*}{U} 7 \left(\frac{y}{R}\right)^{6/7} \sqrt{1 - y/R}\end{aligned}$$

[2]

$$\begin{aligned}\frac{u}{V} &= \sin \frac{\pi y}{2\delta}, \quad \frac{y}{\delta} = \eta, \quad dy = \delta d\eta \\ \delta^* &= \int_0^\delta \left(1 - \sin \frac{\pi y}{2\delta}\right) dy = \delta \int_0^1 (1 - \sin \frac{\pi}{2} \eta) d\eta \\ &= \delta \left(\eta + \frac{2}{\pi} \cos \eta\right) \Big|_0^1 = \delta \left(1 + 0 - 0 - \frac{2}{\pi}\right) = 0.363\delta \\ \theta &= \delta \int_0^1 \sin \frac{\pi}{2} \eta (1 - \sin \frac{\pi}{2} \eta) d\eta \\ &= \delta \left[-\frac{2}{\pi} \cos \eta - \frac{1}{\pi} \left(\frac{\pi}{2} - \sin \frac{\pi}{2} \cos \frac{\pi}{2}\right)\right] \Big|_0^1 = \delta \left(\frac{2}{\pi} - \frac{1}{2}\right) = 0.137\delta \\ H &= \frac{\delta^*}{\theta} = 2.65\end{aligned}$$

[3]

$$\begin{aligned}\frac{v}{V} &= \frac{3}{2}\eta - \frac{1}{2}\eta^3, \quad \frac{y}{\delta} = \eta, \quad dy = \delta d\eta \\ \delta^* &= \int_0^\delta \left(1 - \frac{v}{V}\right) dy = \delta \int_0^1 \left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right) d\eta = \frac{3}{8}\delta = 0.375\delta \\ \theta &= \delta \int_0^1 \left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right) \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right) d\eta = 0.139\delta \\ H &= \frac{\delta^*}{\theta} = 2.69\end{aligned}$$